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MATHEMATICAL SOCIETY.

CONTINUATION OF THE BULLETIN OF THE NEW YORK MATHEMATICAL SOCIETY.

A HISTORICAL AND CRITICAL REVIEW OF MATHEMATICAL SCIENCE.

EDITED BY

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BULLETIN OF THE

AMERICAN MATHEMATICAL SOCIETY

THE SECOND SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Second Summer Meeting of the American Mathematical Society was held in the High School building at Springfield, Mass., on August 27 and 28. The society enjoyed the co-operation of the American Association for the Advancement of Science, the regular sessions of which began on August 29, and the members of the society were entitled to all the privileges extended to the members of the latter organization. Among those present were: Prof. A. L. Baker, Dr. J. H. Boyd, Prof. C. H. Chandler, Prof. L. L. Conant, Prof. C. L. Doolittle, Prof. W. P. Durfee, Prof. E. Frisby, Dr. G. D. Gable, Dr. J. W. L. Glaisher, Prof. A. Hall, Jr., Prof. Ellen Hayes, Dr. G. W. Hill, Prof. W. Woolsey Johnson, Mr. P. A. Lambert, Prof. J. McMahon, Prof. M. Merriman, Prof. F. Morley, Prof. H. B. Newson, Prof. W. F. Osgood, Prof. M. I. Pupin, Mr. R. A. Roberts, Mr. C. H. Rockwell, Prof. J. B. Shaw, Prof. W. E. Story, Prof. H. Taber, Prof. J. M. Van Vleck, Prof. E. B. Van Vleck, Prof. C. A. Waldo, Prof. H. S. White, Prof. J. M. Willard, Prof. C. B. Williams, Prof. F. S. Woods, and Prof. R. S. Woodward.

The President, Dr. G. W. Hill, occupied the chair. Two sessions were held each day, meeting respectively at 10 A.M. and 2.30 P.M. On the recommendation of the Council, Dr. Thomas Jefferson J. See, University of Chicago, Chicago, Ill., was elected to membership. Three nominations for membership were received.

The following papers were read:

1. *The periodic solution as a first approximation in the lunar theory.* Dr. G. W. HILL.

2. *The linear vector operator of quaternions.* Professor J. B. SHAW.

3. *A new application of quaternions to geometry.* Professor J. B. SHAW.

4. *On a generalization of Weierstrass's equation with three terms.* Professor F. MORLEY.

5. *Formulas for the sides of rational plane triangles.* Dr. ARTEMAS MARTIN.

6. *Partial linear transformations of ternary quantics and their concomitants.* Professor J. MCMAHON.

7. *An introduction to the integrell calculus.* Professor W. H. ECHOLS.

8. *On the expansion of a uniform function of a real variable without use of derivatives.* Professor W. H. ECHOLS.

9. *On continuous functions without differential coefficients.* Mr. P. A. LAMBERT.

10. *Concerning Jordan's linear substitution groups.* Professor E. H. MOORE.

11. *Algebraic symbols and* $\sqrt{-1}$. Professor A. L. BAKER.

12. *An application of the method of conformal representation to the study of related differential equations.* Professor E. B. VAN VLECK.

13. *On the differential equations of certain systems of conics.* Mr. R. A. ROBERTS.

14. *On bilinear forms.* Professor H. TABER.

15. *Elementary proof of the quaternion associative principle.* Professor A. S. HATHAWAY.

16. *Asymptotic lines on a circular ring.* Professor H. MASCHKE.

Dr. Hill's paper, since its presentation, has been published in the *Astronomical Journal*, No. 353, September 7, 1895, pp. 137–143. Its object is to obtain the values of the coefficients of the periodic inequalities having the multiples of the mean angular distance of the Moon from the Sun as arguments, when the inclination of the lunar orbit and the two eccentricities are neglected. It is very desirable to have these coefficients with a high degree of accuracy in order to effect their useful employment in the further determination of the motion of the perigee and node, and in fact of all the other coefficients of the periodic inequalities. This work has been done previously by the author in the *American Journal of Mathematics*, vol. 1, but in neglecting the lunar mass and the solar parallax. Mr. Ernest W. Brown has in the same journal supplemented these researches, but still leaving out of consideration the mass of the Moon.

Professor Shaw's first paper is a development of the linear vector operator of quaternions in the form of a scalar part and two vector parts, also as a tensor and a versor, and finally as a sum of nine operators. The forms are similar to those obtained in the articles of Professor Taber (*Amer. Jour. Math.*, vols. 12, 13), but are here developed entirely from quaternion expressions and not from matrices. It was stated that this

paper would be offered to the *American Journal of Mathematics* for publication.

Professor Shaw's second paper applies the quaternion calculus to homogeneous geometry of two dimensions. In the expression $\zeta = xi + yj + zk$, x, y, z are proportional to the areas PBC, PCA, PAB respectively where the point P is referred to the fundamental triangle ABC. By this convention propositions of projective geometry are easily proved, and especially such propositions of modern geometry as those of the Lemoine-Brocard type. The author expected to contribute this paper to the *Annals of Mathematics*.

Professor Morley's paper is published in the present number of the BULLETIN. It contains a simple generalization of the formula

$$\begin{aligned} &\sigma(u+u_1)\ \sigma(u-u_1)\ \sigma(u_2+u_3)\ \sigma(u_2-u_3) \\ +\ &\sigma(u+u_2)\ \sigma(u-u_2)\ \sigma(u_3+u_1)\ \sigma(u_3-u_1) \\ +\ &\sigma(u+u_3)\ \sigma(u-u_3)\ \sigma(u_1+u_2)\ \sigma(u_1-u_2) = 0. \end{aligned}$$

In Dr. Martin's paper a large number of formulæ are deduced for calculating rational numbers which represent the sides of triangles having a rational area. Among them are, for the case of a right-angled triangle,

$$x = 2pq, \qquad y = p^2 - q^2, \qquad z = p^2 + q^2,$$

and, for the case of an oblique triangle,

$$\begin{aligned} x &= (p^2+q^2)(r^2-s^2), \\ y &= 2rs(p^2+q^2) \pm 2rs^2(p^2-q^2), \\ z &= (p^2+q^2)(r^2+s^2) \pm 2rs(p^2-q^2), \end{aligned}$$

in which x, y, z denote the sides, and p, q, r, s any entire numbers whatever. The paper, which contains many numerical applications, will be printed in the *Mathematical Magazine*. In the absence of Dr. Martin his paper was read by the Secretary.

Semi-invariants of a ternary quantic satisfy some, but not all, of the six differential equations which characterize invariants. Professor McMahon's paper shows how to distinguish between those semi-invariants which are the sources of covariants and those which are the sources of semi-covariants, and gives a simple method of deriving from the latter all the coefficients of the semi-covariant, or of a semi-contravariant if desired. A systematic geometrical interpretation of these three kinds of semi-concomitants is presented and appears to be particularly useful in Cartesian co-ordinates. This paper is intended for the *Annals of Mathematics*.

In the absence of Professor Echols his two papers were

presented by the Secretary. The first one was the second section of an essay "On the differell and integrell calculus," the preceding section having been read at the May meeting of the Society. Professor Echols approaches the infinitesimal calculus from the calculus of finite differences. He considers the latter, however, in a greatly generalized form. A differell is defined to be the limit of a ratio whose terms are the nth differences of the function and the independent variable. An integrell is a differell of negative index. The applications presented consisted chiefly in the expansion of functions in terms of differells and integrells. In the second paper some of the more novel results were translated into the language of ordinary calculus.

Mr. Lambert attempted to show that functions of which Weierstrass's derivativeless function is the type may be so considered geometrically that their curves will have determinate tangents. In order to secure agreement of the analytic result with the geometric he found it necessary to replace Weierstrass's supposition in regard to h

$$\tfrac{1}{2}\frac{\pi}{a^n} < h < \tfrac{3}{2}\frac{\pi}{a^n},$$

by the limitation

$$0 < h < \epsilon\frac{\pi}{a^{2n}},$$

n increasing indefinitely, and ϵ being any finite quantity.

In Professor Moore's paper a tactical configuration is established which exactly defines Jordan's linear substitution group when taken fractionally. This configuration is self-reciprocal. The properties of this configuration and of other allied configurations similarly related to certain important subgroups of the main group are developed. Professor Moore's paper will appear in the BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY. It was presented at the meeting by the Secretary.

Professor Baker's paper contained a detailed discussion of the character of the operations of algebra and the theory of imaginary quantities. It will be published in the *American Journal of Mathematics.*

In Professor Van Vleck's paper an aggregate of regular linear differential equations of the second order is taken, each of which has a polynomial solution. The requirement is made that these equations shall have a common group arising from their four common branch points, or in Riemann's phraseology, that they shall be related. This requirement necessitates the introduction of one accessory branch point into each equation. These accessory points do not, however, give rise

to any substitutions of the group. The paper outlines a method by which the position of the accessory points may be investigated, as well as the distribution of the roots of the various polynomials between the four (real) branch points and in the imaginary domain. One result is the determination of the distribution of the roots of all polynomials satisfying differential equations of the second order with exactly four branch points which with their exponent differences are given.

Mr. Roberts' paper gives the differential equations of certain systems of conics in a plane, viz., conics having double contact with two fixed conics, etc., and in space, of conics touching six fixed planes, conics having double contact with three quadrics inscribed in the same developable, circles having double contact with two confocal quadrics, etc. These results are principally deduced by means of elliptic integrals and the first-class of hyper-elliptic integrals, and from them flow certain theorems concerning doubly infinite porisms of curvilinear polygons. The paper appears in the present number of the BULLETIN.

Professor Taber's paper contains a theorem concerning the transformation into itself by an orthogonal substitution of an alternate bilinear form with cogredient variables, and treats of the consequences of this theorem. Thus, denoting by G the group of linear substitutions which transforms into itself an alternate bilinear form with cogredient variables, the theorem in question is that every orthogonal substitution of the group G is the second power of a substitution of this group. From this theorem may be derived: (*a*) The conditions necessary and sufficient that a given substitution of the group G may be generated by the repetition of an infinitesimal substitution of the group. (*b*) A proof that every substitution of the group G can be obtained by the composition of three linear substitutions given by Cayley's rational representation for the substitutions of this group. (*c*) A representation of the coefficients of any orthogonal substitution of the group G as rational functions of the minimum number of parameters, provided the alternate bilinear form is orthogonal. Further it follows from the theorem that every linear substitution which transforms into itself a general bilinear form F is the second power of such a substitution, and can be generated by the repetition of an infinitesimal substitution which also transforms F into itself, provided, if F' denotes the form conjugate to F, that the forms $F+F'$ and $F-F'$ are both of non-evanescent determinant. The paper also contains a solution of the problem to determine the linear transformations between two alternate bilinear forms, depending upon an algebraic equation of degree equal to the number of variables of the forms. The paper will be contrib-

uted to the *Quarterly Journal of Pure and Applied Mathematics.*

In Professor Hathaway's paper the proof consists in identifying, by means of elementary geometry, the product of several versors with the composition of a set of rotations through angles double those of the corresponding versors. The obvious associative principle of the composition of the rotations proves the corresponding associative principle of multiplication of versors. This paper will appear in the BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY. It was presented to the Society by Professor Shaw.

Professor Maschke's paper contains a very elegant application of elliptic functions to curves drawn on the surface of a circular ring. This paper appears in the present number of the BULLETIN. It was presented to the Society by Professor Morley.

At the afternoon session, August 28th, two topics were presented to the Society for general discussion:

(1) "A general subject catalogue or index of mathematical literature."

(2) "The mathematical curriculum of the college and scientific school."

The first discussion was opened by the Secretary, who gave a brief account of the "*Répertoire bibliographique des sciences mathématiques,*" in course of publication by the Mathematical Society of France. The discussion was continued by Professors Morley, Woodward, and McMahon. On motion by Professor McMahon, it was resolved that the Council be requested to consider the desirability of offering to the Mathematical Society of France the coöperation of this Society and of drawing up a plan for such coöperation.

The second discussion was opened by Professor Shaw, who presented a table of statistics showing the character of the mathematical instruction in 101 representative colleges and scientific schools. The discussion was continued by Professors White, Morley, Van Vleck, Doolittle, Chandler, Pupin, and Woodward. It seemed to be generally held that the work of the preparatory schools as a whole is not sufficiently thorough to serve as a satisfactory basis for collegiate courses, that a greater proportion of the students' time should be given to mathematical study, that greater stress should be laid on the fundamental subjects, that elementary portions of applied mathematics should be earlier introduced and more extensively taught, and that spherical trigonometry should be in great part, or altogether, dropped from the required curriculum.

At the close of the discussion the thanks of the Society were tendered to the Springfield Local Committee for the

accommodations and hospitality which the Society had enjoyed, and the meeting was adjourned.

THOMAS S. FISKE.

COLUMBIA COLLEGE, *September* 10, 1895.

THE MEETING OF THE AMERICAN ASSOCIATION.

THE forty-fourth annual meeting of the American Association for the Advancement of Science was held at Springfield, Mass., August 29–September 4, President E. W. Morley, of Cleveland, Ohio, presiding. The attendance was not large, only 367 members being registered. This is 110 less than the number attending the Brooklyn meeting last year. There were 113 new members and 58 fellows elected this year.

Excursions were given to Forest Park, Springfield; to Amherst, Northampton, and Mount Holyoke College, and to Holyoke.

The officers of Section A were as follows: Edgar Frisby, Washington, D. C., Vice-President; Asaph Hall, Jr., Ann Arbor, Mich., Secretary; C. A. Waldo, Greencastle, Ind., Councilor; J. E. Kershner, Lancaster City, Pa.; C. A. Doolittle, Philadelphia, Pa.; L. L. Conant, Worcester, Mass.; C. H. Rockwell, Tarrytown, N. Y., together with the Vice-President and Secretary, sectional committee; J. R. Eastman, Washington, D. C.; S. C. Chandler, Cambridge, Mass.; J. M. Van Vleck, Middletown, Conn., with the Vice-President and Secretary, committee to nominate officers of the section. A. N. Skinner, D. C., Press Secretary.

The work of the Section was devoted almost wholly to astronomy, only two papers in pure mathematics being presented. The tendency of the Section to devote itself to the former class of work was commented on and deplored, and it was hoped that this tendency might be arrested before it had gone too far, and the balance between mathematical and astronomical work restored at the next meeting. This feeling manifested itself to a certain extent in the choice of officers, for the ensuing year, mentioned below.

The papers presented to Section A were as follows:

1. *Development of some useful quaternion expressions, with applications to geometry of three and four dimensions.* By JAMES BYRNIE SHAW, Jacksonville, Ill.

2. *The constant of aberration.* By C. L. DOOLITTLE, Philadelphia, Pa.

3. *On the constant of nutation.* By S. C. CHANDLER, Cambridge, Mass.

4. *Progress of the zone-work at the Naval Observatory, Washington, D. C.* By A. N. SKINNER, Washington, D. C.

5. *Sun-spots and magnetic storms.* By M. A. VEEDER, Lyons, N. Y.

6. *Notes on square numbers whose sum is either a square or the sum of other squares.* By ARTEMAS MARTIN, Washington, D. C. Read by the Secretary.

7. *A convenient formula for computing times of moon rising.* By EDGAR FRISBY, Washington, D. C.

8. *On a slide scale for computing precession.* By EDGAR FRISBY, Washington, D. C.

9. *Some results for stellar parallax from meridian transit observations at the Washburn Observatory.* By A. S. FLINT, Madison, Wis.

10. *The spectrum of β Lyræ.* By E. B. FROST, Hanover, N. H. Read by the Secretary.

11. *Chronology and ancient eclipses.* By S. W. BALCH, Yonkers, N. Y.

12. *Period of R Comæ.* By H. M. PARKHURST, Brooklyn, N. Y.

13. *On the distribution and the secular variation of terrestrial magnetism.* By L. A. BAUER, Washington, D. C. By title.

14. *On making astronomy popular.* By Miss MARY PROCTER. By title.

No. 1 was a development of the A-process in quaternions, where A is defined by the equation $A\,.\,pq = \frac{1}{2}(pq - qp)$. This paper developed the four alternating functions

$$A\,.\,pq = \tfrac{1}{2}(pq - qp),$$

$$S\,.\,pAqr,$$

$$A\,.\,pqr = S\,.\,pAqr - Sp\,.\,A\,.\,qr - Sq\,.\,Arp - Sr\,.\,Apq,$$

$$S\,.\,pAqrs.$$

A set of four quaternions related to one another is deduced, analogous to a set of three rectangular unit-vectors. Various collections of formulas are deduced.

Affixing one of the set of four quaternions to each vertex of a tetrahedron, and letting $P = x_1 l_1 + x_2 l_2 + x_3 l_3 + x_4 l_4$ be that point for which the volumes $PABC$, $PBCD$, $PCDA$, $PDAB$ are as $x_1 : x_2 : x_3 : x_4$, we are enabled to treat solid geometry projectively.

No. 2 was a description of the method employed by Professor Doolittle in determining the constant of aberration. For this purpose a series of 2796 zenith telescope observations were made during the interval from Oct. 10, 1892, to Dec. 27, 1893. Of these observations, 1744 were made before mid-

night, and 1052 after midnight. Each observation gives an equation of the form

$$\phi_0 + k + Ax + By + Cz + Du + E\rho + \tau\mu = \phi$$

where x, y, z, u depend upon the two terms of the latitude variation, ρ is the correction to the assumed value of the constant of aberration, and μ the secular change of the latitude. These equations were combined so as to eliminate ϕ_0 and k, leaving 6 unknown quantities. A least-square solution gives for the constant of aberation $20''.552 \pm .009$.

No. 3 consisted of (*a*) a new determination of the constant of lunar nutation from an extensive body of observations, not hitherto employed for this purpose, and calculated by its distribution to avoid constant sources of error affecting many previous determinations; (*b*) a combination of this value with those deduced by previous investigations, to determine the most probable value of this fundamental constant of astronomy.

No. 4 detailed the relation of that part of the work at the Naval Observatory at Washington under Mr. Skinner's personal direction, to the general scheme of the zone-work planned by the German Astronomical Society. A description was given of the methods employed and the instruments used in this work, and mention was made of the difficulties encountered and of the methods of overcoming them as the work progressed.

No. 5 exhibited printed tables, not heretofore published, illustrating methods of recording sun-spots and magnetic phenomena so as to bring out their relations to each other, and the periodicities involved. Dr. Veeder believed that the records of magnetic observatories might be put in form so as to show the extent of the daily disturbances in such a manner that they could be printed and made readily accessible to the public.

No. 6 developed theorems relating to

(1) n square numbers whose sum is a square.

(2) Three square numbers the sum of whose squares is a square.

(3) Five square numbers the sum of whose squares is a square.

(4) n square numbers the sum of whose squares is a square.

(5) Three square numbers whose sum is equal to the sum of two other squares.

(6) Four square numbers whose sum is equal to the sum of two other squares.

(7) Four square numbers whose sum is equal to the sum of three other squares.

(8) m square numbers whose sum is equal to the sum of n other squares.

(9) $S(1^2) + S(2^2) + S(3^2) + S(4^2) + \ldots\ldots + S(n^2) = \square$, where $S(x^2) = 1^2 + 2^2 + 3^2 + \ldots\ldots + x^2$.

(10) Three square numbers, the sum of every two of which is a square.

(11) Four square numbers, the sum of every three of which is a square.

This paper will be published in the *Mathematical Magazine.*

No. 9 detailed the method of observation used at the Washburn Observatory, which is the same as that employed at the Leyden Observatory, 1885–87, by Prof. J. C. Kapteyn. It consists in observing the differences in the time of meridian transit between the parallax star and two companion stars, one preceding and one following. A list of 76 stars was observed, the Repsold meridian circle being used. These observations included all stars known to have a proper motion of 1″ or more; and some rapid binaries and stars of the first and second magnitude were added later. The results obtained agree with those obtained by previous observers. This paper is to be published in the *Astronomical Journal.*

No. 10 gave a brief account of some of the problems regarding the spectrum of β Lyræ, which have been brought to light by the recent spectroscopic observations of this star at Pulkowa, Potsdam, and London.

No. 11 contained some original researches relating to the bearing on astronomical computation of certain irregularities in the Roman calendar. In astronomical calculations which extend back to early periods of history, it is customary to follow the Old Style calendar. When astronomical events thus computed are also dated in the historical record it is found that the record and computation agree as to the day of the month, provided the date is subsequent to March 1 of 1 B.C. Prior to that date the Roman calendar was more or less irregular, but it has not been customary to follow these irregularities in astronomical work. Instead, the Old Style calendar has been extended backward as far as necessary for calculation; and is, to that extent, fictitious. The actual calendar as observed by the Romans differs from the fictitious or astronomical calendar by amounts varying according to the time. It is important in historical comparisons to know what this difference is; and this was shown in tables prepared for the purpose by Mr. Balch. History indicates that Cæsar began the calendar with a new moon. Computation shows that in 46 B.C. the moon was new on January 2 of the Old Style calendar. Mr. Balch's table showed that the kalends of January as observed coincided with the day preceding January 1 of 46 B.C. The new moon occurred, therefore, two days after the calendar began.

Hence Cæsar's astronomer, Sosogenes, was so much out in his reckoning; but the people doubtless did not observe it, supposing that the new undertaking was begun with the new moon, in accordance with their superstitions.

No. 12 gave the results of observations from 1831 to 1895, on R Comæ, from which it is evident that the period of this star, which was formerly 361 days, has become 365 days; and that, if the period continues to lengthen, the maxima will soon cease to be observable. If, however, we adopt a sine formula, equally well representing all the observations, we find that the period will shorten, bringing the maximum periods out of the twilight and rendering them observable. But if the period should continue to lengthen, the maximum will not be visible again for many years. Either of these theories accords perfectly well with recorded observations; but the former, depending as it does on the sine formula, seems now the more probable.

The next annual meeting of the Association will be held at Buffalo. Instead of beginning in the middle of the week, as has been the custom during the past few years, the sessions will begin on Monday and continue until Friday night. No excursions are to be given until after the close of the meeting. The President for the ensuing year will be E. D. Cope, of Philadelphia.

The officers of Section A for 1896 are as follows: Vice-President, W. E. Story, of Worcester, Mass., and Secretary, E. B. Frost, Hanover, N. H. LEVI L. CONANT.

WORCESTER, MASS., *September* 14, 1895.

ON THE DIFFERENTIAL EQUATIONS OF CERTAIN SYSTEMS OF CONICS.*

BY MR. R. A. ROBERTS.

IN a few cases certain systems of curves can be represented very simply by differential equations of the first order. The latter, since they do not involve the parameter, give readily the tangents to curves of the system, at an arbitrary point of space or of the surface to which the curves are confined. Hence certain loci connected with the system and various properties of the curves can be obtained without having recourse to the integral. A remarkable example of this is to be found in the case of the geodesics on a quadric, all whose

* Read before the AMERICAN MATHEMATICAL SOCIETY at its Second Summer Meeting, Springfield, Mass., August 28, 1895.

properties are obtained by means of the differential equation of the first order, without having recourse to the transcendental final integral.

A short review of the kind of differential equations that I propose to investigate may be of interest.

Considering the complex variable $z_1 = x + iy$, $z_2 = x - iy$ in a plane, we have several remarkable systems comprised in the form

$$\phi(z_1)dz_1 + \phi(z_2)dz_2 = 0.$$

Thus

$$\frac{dz_1}{z_1} \pm \frac{dz_2}{z_2} = 0$$

represents concentric circles and their radii;

$$\frac{dz_1}{\sqrt{(c^2 - z_1^2)}} \pm \frac{dz_2}{\sqrt{(c^2 - z_2^2)}} = 0$$

represents confocal conics, the property that the tangents bisect the angle between the focal radii immediately following from this equation;

$$\frac{dz_1}{\sqrt{Z_1}} \pm \frac{dz_2}{\sqrt{Z_2}} = 0$$

represents confocal Cartesian ovals, where

$$Z = (z - a)(z - b)(z - c);$$

and

$$\frac{mdz_1}{\sqrt{Z_1}} \pm \frac{ndz_2}{\sqrt{Z_2}} = 0$$

represents confocal bicircular quartics, if Z_1 and Z_2 are biquadratic functions of z_1, z_2 respectively, whose roots are homographically connected, and m, n are special functions of the roots. These results were, I believe, first noticed by Darboux.

Further, I may notice that

$$\frac{dz_1}{(Z_1)^{\frac{2}{3}}} + \frac{dz_2}{(Z_2)^{\frac{2}{3}}} = 0,$$

where $Z = (z-a)(z-b)(z-c)$ represents a system of curves of the sixth order, such that the three of the system passing through a given point cut at angles of 120°. I have investigated several properties of this system in an article contributed to vol. 15 of the *Messenger of Mathematics*.

Other remarkable differential equations are those of systems of lines in elliptic co-ordinates; namely, if p, q are the roots of

$$\frac{x^2}{a+p}+\frac{y^2}{b+p}=1,$$

then

$$\frac{dp}{\sqrt{\{(a+p)(b+p)(\alpha+p)\}}}\pm\frac{dq}{\sqrt{\{(a+q)(b+q)(\alpha+q)\}}}=0$$

represents all the lines touching the conic $p+\alpha=0$; and further, if p, q, r are the roots of

$$\frac{x^2}{a+p}+\frac{y^2}{b+p}+\frac{z^2}{c+p}=1,$$

then

$$\Sigma\frac{dp}{\sqrt{\{(a+p)(b+p)(c+p)(\alpha+p)(\alpha'+p)\}}}=0,$$

$$\Sigma\frac{pdp}{\sqrt{\{(a+p)(b+p)(c+p)(\alpha+p)(\alpha'+p)\}}}=0,$$

represent all the lines touching the two confocals $p+\alpha=0$, $p+\alpha'=0$.

The latter equations involve the first class of hyperelliptic integrals, and thus demonstrate the fact of equivalent algebraic relations. These equations, originally due to Liouville, were seen by Haedenkamp to hold for n dimensions. The latter thus deduced the algebraic integrals of the whole class of hyperelliptic integrals.

Another system of differential equations may be found by using the parameters of the two tangents drawn from a point to a conic as co-ordinates. Thus if μ_1, μ_2 are the roots of $\mu^2x-2\mu z+y=0$, a tangent of $z^2-xy=0$,

$$\frac{d\mu_1}{\sqrt{M_1}}\pm\frac{d\mu_2}{\sqrt{M_2}}=0,$$

where $M=(\mu-a)(\mu-b)(\mu-c)(\mu-d)$, represents the system of conics touching the four tangents of $z^2-xy=0$, whose parameters are a, b, c, d. Darboux has made an extensive use of this kind of co-ordinates.

Now I propose to consider here a few forms of differential equations, similar to those just given, which represent systems of conics both in a plane and in space.

First of all, in a plane, I consider the three systems of conics having double contact with two fixed conics. Let the

latter be referred to their common self-conjugate triangle, and let us consider the system of conics (touching four fixed lines)

$$U_p = \frac{x^2}{a+p} + \frac{y^2}{b+p} + \frac{z^2}{c+p} = 0; \qquad (1)$$

then if p, q are the roots of this equation, the differential equation

$$\frac{dp}{\sqrt{[(a+p)(b+p)(\alpha+p)(\alpha'+p)]}} \pm \frac{dq}{\sqrt{[(a+q)(b+q)(\alpha+q)(\alpha'+q)]}} = 0 \qquad (2)$$

represents a system of conics having double contact with the conics obtained by putting $\alpha + p = 0$ and $\alpha' + p = 0$ in (1). For, by the theory of elliptic functions, the integral of the equation can be written as a linear relation between any three of the quantities $\sqrt{\{(a+p)(a+q)\}}$, $\sqrt{\{(b+p)(b+q)\}}$, $\sqrt{\{(\alpha+p)(\alpha+q)\}}$, $\sqrt{\{(\alpha'+p)(\alpha'+q)\}}$; but the first two of these quantities are proportional to x, y, respectively, and the last two to $\sqrt{U_\alpha}$ and $\sqrt{U_\alpha'}$. Thus we get the equivalent equations

$$lx + my + n\sqrt{U_\alpha} = 0,$$
$$l'x + m'y + n'\sqrt{U_\alpha'} = 0,$$

which represent a conic having double contact with U_α and U_α', respectively.

Again, by using the factors $(b+p)(c+p)$ and $(a+p)(c+p)$ successively, instead of $(a+p)(b+p)$ in (2), we get the differential equations of the two other systems of conics.

As a particular case of this result, suppose we consider the ordinary plane elliptic co-ordinates, viz., those determined by

$$\frac{x^2}{\mu^2} + \frac{y^2}{\mu^2 - c^2} = 1, \qquad \frac{x^2}{\nu^2} - \frac{y^2}{c^2 - \nu^2} = 1;$$

then

$$\left.\begin{aligned}
\frac{d\mu}{\sqrt{[(\mu^2-a^2)(\mu^2-a'^2)]}} \pm \frac{d\nu}{\sqrt{[(a^2-\nu^2)(a'^2-\nu^2)]}} &= 0, \\
\frac{d\mu}{\sqrt{[(\mu^2-a^2)(\mu^2-a'^2)(\mu^2-c^2)]}} \pm \frac{d\nu}{\sqrt{[(a^2-\nu^2)(\nu^2-a'^2)(c^2-\nu^2)]}} &= 0, \\
\frac{\mu d\mu}{\sqrt{[(\mu^2-a^2)(\mu^2-a'^2)(\mu^2-c^2)]}} \pm \frac{\nu d\nu}{\sqrt{[(a^2-\nu^2)(\nu^2-a'^2)(c^2-\nu^2)]}} &= 0,
\end{aligned}\right\} \qquad (3)$$

represent the three systems of conics having double contact with two confocal conics.

If we make $a' = \infty$, which corresponds to the two imaginary points at infinity, we get

$$\left.\begin{aligned} \frac{d\mu}{\sqrt{(a^2-\mu^2)}} \pm \frac{d\nu}{\sqrt{(a^2-\nu^2)}} &= 0, \\ \frac{\mu d\mu}{\sqrt{\{(\mu^2-a^2)(\mu^2-c^2)\}}} \pm \frac{\nu d\nu}{\sqrt{\{(a^2-\nu^2)(c^2-\nu^2)\}}} &= 0, \end{aligned}\right\} \quad (4)$$

as the differential equations of the two systems of circles having double contact with the conic $\mu = a$. The third equation gives in the same case the system of tangents to $\mu = a$.

Proceeding now to the three dimensions, let us consider the system of quadrics (inscribed in the same developable)

$$U_p = \frac{x^2}{a+p} + \frac{y^2}{b+p} + \frac{z^2}{c+p} + \frac{u}{d+p} = 0; \quad (5)$$

then, if p, q, r are the roots of this equation,

$$\left.\begin{aligned} \Sigma \frac{dp}{\sqrt{\{(a+p)(b+p)(c+p)(\alpha+p)(\alpha'+p)(\alpha''+p)\}}} &= 0, \\ \Sigma \frac{pdp}{\sqrt{\{(a+p)(b+p)(c+p)(\alpha+p)(\alpha'+p)(\alpha''+p)\}}} &= 0, \end{aligned}\right\} \quad (6)$$

represents a conic having double contact with the three quadrics U_a, U_a', U_a'', for by the theory of the first class of hyperelliptic integrals the integrals can be expressed as linear relations between any three of the six quantities

$$\begin{aligned} &\sqrt{\{(a+p)(a+q)(a+r)\}}, \quad &&\sqrt{\{(\alpha-p)(\alpha-q)(\alpha-r)\}}, \\ &\sqrt{\{(c+p)(c+q)(c+r)\}}, \quad &&\sqrt{\{(\alpha'-p)(\alpha'-q)(\alpha'-r)\}}, \\ &\sqrt{\{(b+p)(b+q)(b+r)\}}, \quad &&\sqrt{\{(\alpha''-p)(\alpha''-q)(\alpha''-r)\}}. \end{aligned}$$

Now the first three of these are proportional to x, y, z, respectively, and the three last are proportional to $\sqrt{U_a}$, $\sqrt{U_a'}$, $\sqrt{U_a''}$. Thus we get

$$\begin{aligned} lx + my + nz &= 0, \\ l'x + m'y + n'\sqrt{U_a} &= 0, \\ l''x + m''y + n''\sqrt{U_a'} &= 0, \\ l'''x + m'''y + n'''\sqrt{U_a''} &= 0, \end{aligned}$$

and these equations obviously represent a conic having double contact with the quadrics U_a, U_a', U_a''.

Again, by interchanging the constants a, b, c, d we obtain three other systems similar to (6).

Thus we have demonstrated that there are four different systems of conics that can be described to have double contact with three quadrics inscribed in the same developable.

As confocal quadrics are of this kind, their equations being of the form

$$\frac{x^2}{a+p}+\frac{y^2}{b+p}+\frac{z^2}{c+p}=1,$$

we obtain the differential equations of conics described to have double contact with three confocal quadrics by making $p=\infty$ in the three equations similar to (6), each of which involves d.

As a special case, suppose $\alpha''=\infty$, which corresponds to the imaginary circle at infinity: then the conics become circles having double contact with two confocal quadrics.

Now equations (6) in this case become simply the differential equations of lines touching the two confocals, but the three similar equations give those of the derived systems, viz.:

$$\Sigma\frac{dp}{\sqrt{\{(a+p)(b+p)(\alpha+p)(\alpha'+p)\}}}=0,$$

$$\Sigma\frac{pdp}{\sqrt{\{(a+p)(b+p)(\alpha+p)(\alpha'+p)\}}}=0,$$

and the two other similar equations obtained by interchanging a, b, c. Thus there are three systems of circles which can be described to have double contact with two confocal quadrics. Furthermore, either of these quadrics can degenerate to a focal conic. Thus, for instance, since $p+a=0$, $p+b=0$ give the two focal conics, we have

$$\Sigma\frac{\pm dp}{(a+p)(b+p)}=0, \qquad \Sigma\frac{\pm pdp}{(a+p)(b+p)}=0;$$

$$\Sigma\frac{dp}{(a+p)\sqrt{\{(b+p)(c+p)\}}}=0, \qquad \Sigma\frac{pdp}{(a+p)\sqrt{\{(b+p)(c+p)\}}}=0;$$

$$\Sigma\frac{dp}{(b+p)\sqrt{\{(a+p)(c+p)\}}}=0, \qquad \Sigma\frac{pdp}{(b+p)\sqrt{\{(a+p)(c+p)\}}}=0;$$

as the differential equations of circles described to intersect twice each of the two focal conics.

In each of the preceding equations we have $dp:dq:dr=A:\pm B:\pm C$, say; that is, there are, in consequence of the double signs, four tangents to the curves of the system that pass through a given point. Thus there are four curves of the system that pass through a point, and from the differ-

ential equations we can find properties of these curves, and loci connected with their intersection. Thus in elliptic co-ordinates, the normals to the confocal quadrics being at right angles to each other, it follows that the tangents to the four conics of the aforementioned systems that pass through a point are equally inclined to the normal of one of the quadrics; that is, if α, β, γ are the angles which one of the tangents makes with the normals of the confocal quadrics, we shall have $\cos\alpha$, $\cos\beta$, $\cos\gamma$ respectively proportional to $\lambda, \pm\mu, \pm\nu$, say, for the tangents of the four conics of the system. Hence, if two of the conics should cut at right angles, we should have $\lambda^2 + \mu^2 = \nu^2$, from which we can derive the equation of the locus in elliptic co-ordinates.

I now proceed to consider the problem: to find the differential equations of the system of conics touching six fixed planes in space.

Now there is a definite unique twisted curve of the third degree which may be described to osculate the six fixed planes. Hence, since an osculating plane of the twisted cubic can be written in the form

$$X_\mu = \mu^3 x + \mu^2 y + \mu z + u = 0,$$

it follows that the six planes can be obtained by substituting six quantities $a_1, a_2, a_3, a_4, a_5, a_6$ in this equation in place of μ. Now let us consider the differential equations

$$\Sigma\frac{d\mu}{\sqrt{M}} = 0, \qquad \Sigma\frac{\mu d\mu}{\sqrt{M}} = 0, \tag{7}$$

where the summation refers to the parameters μ_1, μ_2, μ_3 of the three osculating planes which can be drawn through a point, and

$$M = (\mu - a_1)(\mu - a_2)(\mu - a_3)(\mu - a_4)(\mu - a_5)(\mu - a_6).$$

Then, by the theory of the first class of hyperelliptic integrals, we have as the integrals a series of linear relations connecting the six quantities

$$\sqrt{\{(a_1-\mu_1)(a_1-\mu_2)(a_1-\mu_3)\}}, \quad \sqrt{\{(a_2-\mu_1)(a_2-\mu_2)(a_2-\mu_3)\}}, \quad \ldots, \quad \sqrt{\{(a_6-\mu_1)(a_6-\mu_2)(a_6-\mu_3)\}}.$$

But, from the equation of the osculating plane, the expression $(a_1 - \mu_1)(a_1 - \mu_2)(a_1 - \mu_3)$ is proportional to the osculating plane corresponding to the parameter a_1, and similarly for the other expressions. Thus we have linear relations connecting every three of the radicals $\sqrt{X_{a_1}}, \sqrt{X_{a_2}}, \ldots, \sqrt{X_{a_6}}$;

or we may take each of these proportional to $p_1 + q_1 t$, $p_2 + q_2 t$, . . ., $p_6 + q_6 t$, where t is an arbitrary parameter. Hence we get

$$\lambda X_{a_1} = (p_1 + q_1 t)^2, \quad \lambda X_{a_2} = (p_2 + q_2 t)^2, \quad \ldots, \quad \lambda X_{a_6} = (p_6 + q_6 t)^2.$$

Now these equations obviously represent a curve of the second degree, viz., a conic section, touching the six planes X_{a_1}, X_{a_2}, . . ., X_{a_6}. Consequently the differential equation (7) represents the system of conics described to touch the six planes. Since there are four values of the ratios $d\mu_1 : d\mu_2 : d\mu_3$, it follows that there are four conics of the system passing through an arbitrary point of space.

In a paper contributed to *The Proceedings of the London Mathematical Society*, vol. 18, p. 202, I showed that if certain conditions were satisfied a doubly infinite number of polygons could be circumscribed about two confocal quadrics so as to have their vertices on another confocal quadric. This result was obtained by means of the theory of the first class of hyperelliptic integrals. Similar reasoning could be employed in the case of the conics that I have been considering. Thus, under certain conditions, we could have an infinite number of curvilinear polygons formed by conics having double contact with two confocal quadrics and intersecting on another confocal quadric. There would also be infinite systems of curvilinear polygons in the other cases that I have investigated.

An interesting question arises in connection with the preceding results, viz., as to the reality of the curves of the different systems. Thus, for instance, in the case of two confocal ellipses represented in elliptic co-ordinates by $\mu = a$, $\mu = a'$, the conics belonging to the system

$$\frac{d\mu}{\sqrt{\{(\mu^2 - a^2)(\mu^2 - a'^2)\}}} \pm \frac{d\nu}{\sqrt{\{(a^2 - \nu^2)(a'^2 - \nu^2)\}}} = 0$$

are not real, because μ must obviously lie between a and a', and thus $(\mu^2 - a^2)(\mu^2 - a'^2)$ is negative, while $(a^2 - \nu^2)(a'^2 - \nu^2)$ is positive.

But the two other systems contain real conics, and in other cases there seem to be only two systems with real conics.

A case, however, not already noticed, in which the three systems are real, deserves to be mentioned. One of the fixed conics is, however, imaginary, namely, the imaginary circle at infinity on a sphere. Consider the confocal cones

$$\frac{x^2}{\mu^2} + \frac{y^2}{\mu^2 - b^2} = \frac{z^2}{c^2 - \mu^2},$$

$$\frac{x^2}{\nu^2} - \frac{y^2}{b^2 - \nu^2} = \frac{z^2}{c^2 - \nu^2},$$

intersecting the sphere $x^2+y^2+z^2=1$ in confocal sphero-conics; then the differential equations

$$\frac{d\mu}{\sqrt{\{(\mu^2-b^2)(\mu^2-a^2)\}}}\pm\frac{d\nu}{\sqrt{\{(b^2-\nu^2)(a^2-\nu^2)\}}}=0,$$

$$\frac{d\mu}{\sqrt{\{(c^2-\mu^2)(a^2-\mu^2)\}}}\pm\frac{d\nu}{\sqrt{\{(c^2-\nu^2)(a^2-\nu^2)\}}}=0,$$

$$\frac{\mu d\mu}{\sqrt{\{(\mu^2-b^2)(c^2-\mu^2)(\mu^2-a^2)\}}}\pm\frac{\nu d\nu}{\sqrt{\{(b^2-\nu^2)(c^2-\nu^2)(a^2-\nu^2)\}}}=0,$$

represent the three systems of circles having double contact with the sphero-conic $\mu=a$, and these it is easy to see from geometrical considerations are real. Circles of course may be considered as conics having double contact with $\mu=\infty$, or $x^2+y^2+z^2=0$. By projection then on a plane it would appear that we could have three real systems of conics having double contact with one real conic and one imaginary conic.

It would be interesting also to consider the reality of the conics having double contact with three quadrics. For instance, it does not seem possible to have any real conics having real double contact with three confocal ellipsoids. But I must defer the consideration of this question to another opportunity.

ASYMPTOTIC LINES ON A CIRCULAR RING.*

BY PROFESSOR H. MASCHKE.

THE equation of the asymptotic lines of a tore as well as the expression of the length of the arc of these curves leads to elliptic functions. The formulas appear in a remarkably simple form.†

Assuming the radius CA of the rotating circle $=1$, and denoting the distance CO of its centre from the axis of rotation by λ, the tore is represented by these equations:

$$x=v\cos u,\quad y=v\sin u,\quad z=\sqrt{1-(v-\lambda)^2},$$

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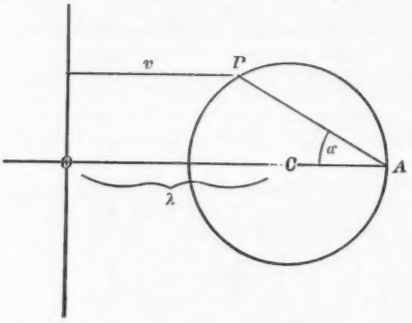
* Read before the AMERICAN MATHEMATICAL SOCIETY at its Second Summer Meeting, Springfield, Mass., August 28, 1895.

† I have not found any literature with regard to this subject excepting a short notice in the *Fortschritte der Mathematik*, 1889, page 765, where reference is made to T. MOTODA, "Asymptotic lines on a circular ring," *Tokio Math. Soc.*, vol. 4, 217–219, a paper which has been inaccessible to me.

where u is the angle between the plane of the revolving circle and the XZ-plane and v the distance of a point P of the surface from the Z-axis, the axis of rotation.



Then the differential equation of the asymptotic curves is this:

$$\left(\frac{dv}{du}\right)^2 = v(\lambda - v)(1 + \lambda - v)(1 - \lambda + v).$$

From this equation we find v in terms of Weierstrass's $\wp$-function

$$v = \frac{2\lambda(6\wp u - 1)}{12\wp u + 1},$$

the invariants of $\wp u$ being

$$g_2 = \frac{1 + 3\lambda^2}{12}, \quad g_3 = \frac{1 - 9\lambda^2}{12.18},$$

and

$$e_1 = \frac{3\lambda - 1}{12}, \quad e_2 = \frac{1}{6}, \quad e_3 = -\frac{3\lambda + 1}{12}.$$

In terms of σ-functions there is:

$$v = \frac{2\lambda\sigma_2{}^2 u}{\sigma_1{}^2 u + \sigma_2{}^2 u},$$

and denoting $\measuredangle OAP = \alpha$,

$$\tan\alpha = \frac{\sigma_1 u}{\sigma_2 u}.$$

The length of the arc of an asymptotic curve is given by the integral

$$s = \sqrt{\lambda} \int_v^\lambda \frac{dv}{\sqrt{(\lambda - v)(1 + \lambda - v)(1 - \lambda + v)}}.$$

Introducing the $\wp$-function with the invariants $g_2 = \frac{1}{4\lambda^2}$, $g_3 = 0$, and $e_1 = \frac{1}{4\lambda}$, $e_2 = 0$, $e_3 = -\frac{1}{4\lambda}$, $\left(k^2 = \frac{e_2 - e_3}{e_1 - e_3} = \frac{1}{2}\right)$, we obtain:

$$\lambda - v = \frac{1}{4\lambda} \cdot \frac{1}{\wp s}.$$

UNIVERSITY OF CHICAGO, *August* 23, 1895.

ON A GENERALIZATION OF WEIERSTRASS'S EQUATION WITH THREE TERMS.

BY PROFESSOR F. MORLEY.

THE expression

$$\prod_{\lambda=1}^{n} \sigma(u - b_\lambda)/\sigma(u - a_\lambda)$$

is an elliptic function of u if

$$\Sigma a_\lambda = \Sigma b_\lambda.$$

The sum of the residues is zero; that is,

$$(1) \qquad \sum_{\lambda=1}^{n} \frac{\sigma(a_\lambda - b_1) \ldots \sigma(a_\lambda - b_\lambda) \ldots \sigma(a_\lambda - b_n)}{\sigma(a_\lambda - a_1) \ldots \quad 1 \quad \ldots \sigma(a_\lambda - a_n)} = 0.$$

Being now only concerned with differences, we can, by a suitable addition to each a and b, write

$$(2) \qquad \Sigma a_\lambda = \Sigma b_\lambda = 0.$$

When $n = 2$, the equation (1) is in no way characteristic of the σ-function, but is true of any odd function.

When $n = 3$, (1) becomes

$$(3) \qquad \begin{aligned} &\sigma(a_1 - b_1)\sigma(a_1 - b_2)\sigma(a_1 - b_3)\sigma(a_2 - a_3) \\ &+ \sigma(a_2 - b_1)\sigma(a_2 - b_2)\sigma(a_2 - b_3)\sigma(a_3 - a_1) \\ &+ \sigma(a_3 - b_1)\sigma(a_3 - b_2)\sigma(a_3 - b_3)\sigma(a_1 - a_2) = 0. \end{aligned}$$

To identify this with the equation of three terms as given by Weierstrass, write

$$a = b_1 - a_1,$$
$$b = a_1 - b_2,$$
$$c = a_1 - b_3,$$
$$d = a_3 - a_2,$$

and use the notation given by Harkness and Morley, Theory of Functions, p. 313.

We have

$$a' = b_1 - a_3, \qquad a'' = b_1 - a_2,$$
$$b' = a_2 - b_2, \qquad b'' = a_2 - b_3,$$
$$c' = a_2 - b_3, \qquad c'' = a_2 - b_3,$$
$$d' = a_1 - a_3, \qquad d'' = a_3 - a_1,$$

so that our equation (3) may be written

$$\sigma a \sigma b \sigma c \sigma d + \sigma a' \sigma b' \sigma c' \sigma d' + \sigma a'' \sigma b'' \sigma c'' \sigma d'' = 0,$$

which is equation (48) of the page referred to.

For higher values of n the equation (1) appears to be the simplest of the various possible extensions of the equation with three terms.

NOTES.

AMONG recent academic appointments are the following: Professor Samuel M. Barton, recently of the Virginia Agricultural and Mechanical College, has been appointed professor of mathematics at the University of the South, Sewanee, Tenn.; Dr. Alexander Macfarlane, formerly professor of physics at the University of Texas, has accepted a lectureship in electrical engineering at Lehigh University, South Bethlehem, Pa.; Dr. E. B. Van Vleck has resigned an instructorship at the University of Wisconsin in order to become associate professor of mathematics at Wesleyan University, Middletown, Conn.; Professor C. A. Waldo has been called from De Pauw University to the chair of mathematics at Purdue University, La Fayette, Ind.; Professor W. J. Hussey, of Stanford University, has been appointed astronomer at the Lick Observatory, to succeed Professor E. E. Barnard, who has been called to a chair of astronomy at the University of Chicago.

DR. PAUL STÄCKEL, of Halle, has been appointed professor extraordinarius at Königsberg, and Professor V. Eberhardt, of Königsberg, has been called to Halle.

WE are grieved to learn of the death of Dr. Ernst Ritter, on August 23, of typhoid fever, at the government hospital

on Ellis Island in New York harbor. He was on his way from Germany to begin his duties as assistant professor of mathematics at Cornell University.

THE STEINER PRIZE.—The Academy of Sciences of Berlin at the Leibnitz Meeting, July 3, 1890, announced the following problem for the Steiner Prize:—The solution of an important problem in the theory of lines of curvature of surfaces; preferably, the determination of the conditions under which the lines of curvature of algebraic surfaces are algebraic curves. No paper on this subject was received. In accordance with the terms of the Steiner foundation, the Academy has utilized the prize, thus unawarded, for the purpose of recognizing certain important geometrical contributions published during the past few years. One half of it has been awarded to Dr. Sigmund Gundelfinger, professor at the polytechnic school at Darmstadt, for his remarkable work leading to the fundamental investigation and development of the methods introduced into geometry by Hesse; the other half to Dr. Friedrich Schottky, professor at the University of Marburg, who has afforded valuable assistance in a series of most important special problems of geometry by demonstrating their relations to the theory of abelian functions of two, three, and four variables.

For the year 1900 the Academy announces the following problem for the Steiner Prize:—To completely solve any important, hitherto unsolved problem relating to the theory of curved surfaces, taking into account so far as possible the methods and principles evolved by Steiner. It is required that sufficient analytical explanations shall accompany the geometrical investigations to verify the correctness and completeness of the solution. Without wishing to limit the choice of subject the Academy takes the opportunity to call attention to the special problems to which Steiner has referred in his general remark at the end of his second paper on maximum and minimum in figures in a plane, on a sphere, and in space. For the solution of the problem a prize of 4000 marks is offered, with an additional sum of 2000 marks. Papers offered in competition may be written in German, French, English, Italian, or Latin, and must be submitted before December 31, 1899. The result will be announced at the Leibnitz Meeting of 1900.

THE attendance at the meeting of the British Association for the Advancement of Science, held at Ipswich, September 11–18, was somewhat smaller than at the Oxford meeting last year. Professor W. M. Hicks was president of Section A, representing mathematics and physics. His address, the sub-

ject of which was "The fluid theories of ether and matter," appears in *Nature* of September 12.

THE French Association for the Advancement of Science met at Bordeaux, August 4, under the presidency of Émile Trélat. There were more than 800 members present. According to *La Nature* the following were among the most interesting mathematical papers read:—Mr. TORRIS, *On an algebraic machine;* Mr. COCCOZ, *On magic squares;* Mr. GUIMARET, *On vortices;* Mr. COLLIGNON, *On certain questions in geometry.*

WE take pleasure in announcing the following work as in press: *Theory of functions of a complex variable, with special reference to Riemann surfaces.* By Dr. H. DURÈGE, late Professor in the University of Prague. Authorized translation from the fourth German edition. By GEORGE EGBERT FISHER, M.A., Ph.D., and ISAAC J. SCHWATT, Ph.D., of the University of Pennsylvania, Philadelphia. The work is being published by the translators.

AMONG the courses for the winter semester at Berlin are the following:—Professor Fuchs: Analytical mechanics; Abelian functions;—Professor Frobenius: Theory of numbers;—Professor Schwarz: Synthetic geometry; Selected chapters from the theory of analytical functions; Theory of elliptic functions, making use of his edition of "Formeln und Lehrsätze zum Gebrauche der elliptischen Functionen";—Professor Knoblauch: Analytical geometry; Definite integrals; Mathematical exercises for younger students;—Professor Hensel: Algebra; Integral calculus; Exercises in algebra;—Professor Hettner: Theory of probabilities;—Dr. Schlesinger: Theory of Fourier's series and spherical harmonics, with applications to questions of mathematical physics; Theory of substitution groups, with applications to linear differential equations;—Dr. Kötter: Geometry of circle and sphere systems, with special reference to bicircular curves and surfaces of the fourth order; Differential calculus and introduction to analysis; Exercises;—Dr. Hoppe: Analytical geometry, in connection with his text-book; Differential calculus and series, in connection with his text-book;—Dr. Wien: Hydrodynamics.

AMONG the courses for the winter semester at Leipsic are the following: Professor Scheibner: On the hypergeometric series and Eulerian integrals;—Professor Neumann: Selected chapters from mathematical physics; Discussion of mathematical problems;—Professor Lie: Introduction to metrical geometry; On differential equations which admit infinitesimal transformations; Exercises in mathematical seminarium;—

Professor Mayer: General introduction to the differential and integral calculus; Exercises;—Professor Engel: Introduction to analytical mechanics; Higher algebra; Conference on mathematical problems;—Dr. Scheffers: Introduction to the theory of linear differential equations; Descriptive geometry accompanied by exercises; Kinematics (theory of mechanisms).

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